

What is this?

This is a puzzle called the Tower of Hanoi, reportedly based on a temple in Hanoi where monks transfer rings from one pole to the others: the world is supposed to end when the job is done.

The rules are simple:

- The aim is to transfer the rings from one pole to one of the others
- Only one ring at a time can be moved
- No ring is allowed to rest on a smaller ring

Here's a starting position



If we number the rings from number 1 (the smallest), the moves are in the order:
ring 1, ring 2, ring 1, 3, 1, 2, 1, 4, 1, 2, 1, 3, 1, 2, 1, 5, 1, 2, etc.

(Ring 1 moves cyclically around the poles every second move starting with the first, and there is only one possible move on the even moves.)

The smallest ring (ring 1) is moved every second move, the next (ring 2) is moved every 4th move, the next (ring 3) is moved every 8th move, etc, and in general ring r is moved every 2^r moves.

Give it a try and see how you go.

The Tower of Hanoi has a natural application to the management of holdings of backups of data and has been used by CSIRO since about 1998.

(In the past individual magnetic tape cartridges were used for backups. The numbers on the rings in this puzzle have been cut from coloured barcode labels used on such cartridges.)

For more information, ask Rob. Bell, or see

https://wiki.hpsc.csiro.au/personal/edw192/backups_summary#local_modifications_to_rsyc and

https://en.wikipedia.org/wiki/Backup_rotation_scheme#Extensions_and_Example, which contains material by Rob Bell on Tower of Hanoi Extensions and Example.

Introduction to backup management

Saving of backups or snapshots of data holdings can help to guard against losses from many sources, e.g. user error, system failure, physical destruction, or cyber intrusion.

Keeping a copy of the data at a range of previous times can be important. A simple mirror of the current holdings does not guard against losses from such events as data corruption, software failures and un-detected intrusion.

There are few installations that can keep all backups forever, and so decisions need to be made about which backups to keep. With backups often done daily, people have typically tried to keep daily backups for a week, weekly backups for a month, monthly backups for 6 months, etc, but this is surprisingly difficult to schedule, and can leave gaps. Intuitively and in our experience, the probability of needing to restore from a backup recedes with the age of the backups.

There is a better way to manage backup holdings, based on the Tower of Hanoi – see https://en.wikipedia.org/wiki/Backup_rotation_scheme#Extensions_and_Example, and in particular the section “1.3.4Extensions and example” written by me.

Here’s a table showing the rings on each pole for the first 16 moves.

Move number	Ring to move	Pole 1	Pole 2	Pole 3
0	–	1-9	–	–
1	1	2-9	1	–
2	2	3-9	1	2
3	1	3-9	–	1,2
4	3	4-9	3	1,2
5	1	1,4-9	3	2
6	2	1,4-9	2,3	–
7	1	4-9	1,2,3	–
8	4	5-9	1,2,3	4
9	1	5-9	2,3	1,4
10	2	2, 5-9	3	1,4
11	1	1,2,5-9	3	4
12	3	1,2,5-9	–	3,4
13	1	2,5-9	1	3,4
14	2	5-9	1	2,3,4
15	1	5-9	–	1,2,3,4
16	5	6-9	5	1,2,3,4

Basic Tower of Hanoi for backups

Assign a sequence or cycle number 0 to the first backup – a special case, to be kept ‘forever’.

Assign a sequence or cycle number, c, to each successive backup.

Assign a set number to each backup, defined as one more than the highest power of two in the factors of the cycle number – see the table below. The set numbers are precisely the ring number that is to be moved on each cycle number in the Tower of Hanoi.

Use each backup every 2 to the power of its 'set number' of cycles. Keep one of each backup for each set.

For example

Cycle number	c	0	1	2	3	4	5	6	7	8	9	10	...
Factors of 2	f	-	2^0	2^1	2^0	2^2	2^0	2^1	2^0	2^3	2^0	2^1	...
Powers of two factors	p	-	0	1	0	2	0	1	0	3	0	1	...
Set number	$S=p+1$	0	1	2	1	3	1	2	1	4	1	2	...
Cycles to be kept		∞	2	4	2	8	2	4	2	16	2	4	...

Here is a table showing the cycle number of the holdings at each stage up to cycle 11

Cycle number	0	1	2	3	4	5	6	7	8	9	10	11
Set 0	0	0	0	0	0	0	0	0	0	0	0	0
Set 1		1	1	3	3	5	5	7	7	9	9	11
Set 2			2	2	2	2	6	6	6	6	10	10
Set 3					4	4	4	4	4	4	4	4
Set 4									8	8	8	8

So, for example, after 11 cycles, we will have the following backups:

Cycle 0 set 0 – to be kept forever

Cycle 4 set 3 – to be kept for 8 cycles

Cycle 8 set 4 – to be kept for 16 cycles

Cycle 10 set 2 – to be kept for 4 cycles.

Cycle 11 set 1 – to be kept for 2 cycles

There is close coverage of the recent backups, with sparser keeping further back in time. The last 2 backups are always kept.

In the original description of Tower of Hanoi management, backups were on numbered tapes, so tape one was used every second time, tape 2 every 4th time, etc. Now, backups are more likely found in directories, and these can be labelled with the cycle or sequence number to make management easy.

Many variants are possible; for example, if you want more coverage at the cost of using more storage, you could keep two or more of each cycle. Here's an example with dates and sequence numbers embedded in directory names, and Tower of Hanoi used to assign set numbers to each backup: the coverage density is set to 1.25, meaning that either one or two of each backup set will be kept, and around one quarter of the sets will have two copies kept.

```
home.20141002.seq.0 set 0
home.20160321.seq.512 set 10
home.20170811.seq.1024 set 11
home.20180426.seq.1280 set 9
home.20180904.seq.1408 set 8
home.20190108.seq.1536 set 10
home.20190312.seq.1600 set 7
home.20190412.seq.1632 set 6
home.20190428.seq.1648 set 5
home.20190514.seq.1664 set 8
home.20190522.seq.1672 set 4
home.20190526.seq.1676 set 3
home.20190530.seq.1680 set 5
home.20190531.seq.1681 set 1
home.20190601.seq.1682 set 2
home.20190602.seq.1683 set 1
```

At this stage, there are two backups of sets 1, 5, 8 and 10 kept, to give greater coverage. These are daily backups, and the latest 4 are kept. The gaps (cycle numbers, but roughly days) between backups, from most recent to the oldest, are:

1, 1, 1, 4, 4, 8, 16, 16, 32, 64, 128, 128, 256, 512 and 512.

The backup coverage spreads out exponentially back in time, which is desirably related to the probability of a restore being needed.

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Robert Bell